An Insightful Theory of Flight

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Abstract and Significance

A relatively simple but comprehensive description of flight in terms of a volume integral of air's density-average acceleration around an aircraft is able to overcome misleading over-simplifications of explanations based on Bernoulli's equation and momentum theory. Applications of the equation indicate that the historic absence of such a valued insight has resulted in a series of engineering design paradigms with significant implications on aircraft fuel economy and capabilities. To achieve efficient flight, a critical part of the design process is to provide surfaces of 0°-4° pitch for transfer of air pressure to lift force on the flying object.

Introduction and Background

Theories of Flight - It is recognized that simplified explanations of flight using Bernoulli's equation and momentum theory are inaccurate.[1, 2] Also, the accurate explanations from Euler's equations can be equally as problematic because they are too removed from practical application. In aircraft design there is a need for usefully simple theorems and heuristics to guide in the design process because these heuristics help identify design options for greater study. This paper is on an integral based theorem that is accurate, can be directly related to aircraft surfaces and streamlines, and suggests alternative approaches to aircraft design.

Explanations based on Bernoulli's equation dictate that air must travel faster to traverse the upper surface of a properly engineered airfoil, and faster velocity results in lower pressure according to the Equation 1 Bernoulli's equation.   This approach typically predicts a low pressure immediately above the leading edge of the wing; however, as illustrated by Figure 1, that section of the wing tends to have pressures higher than ambient pressure.

$$\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g\Delta z = 0 \qquad \text{Equation 1}$$

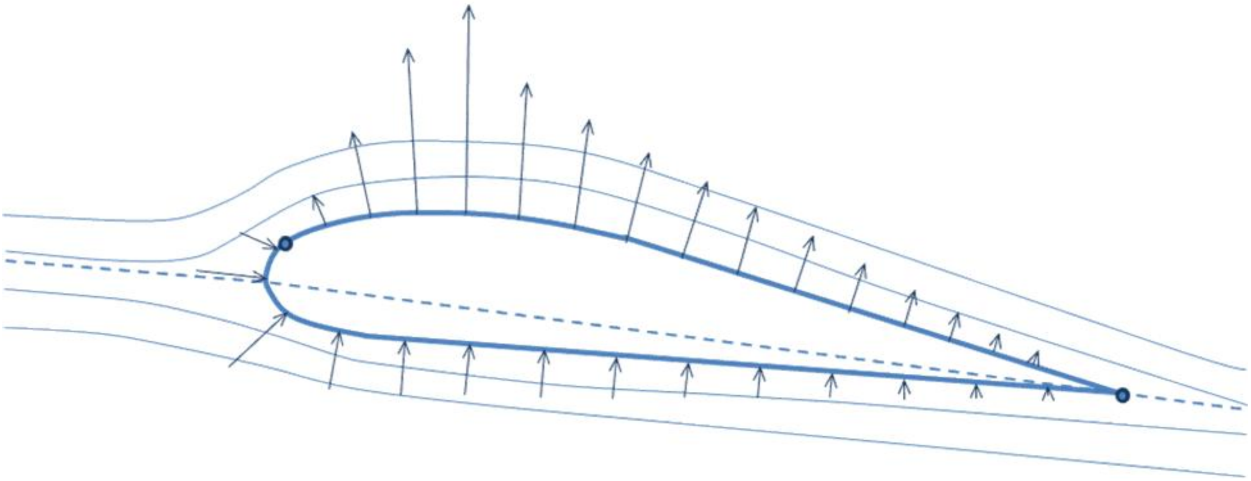

Figure 1. Airfoil with example streamlines and pressure force vectors.

Explanations based on Newton's second law tend to focus on airfoil downwash[3], with the explanation that the downward momentum of the downwash must have an equal and opposite "lifting" momentum on the wing. This type of anecdotal explanation only explains a small part of the bigger picture, and thus is not effective for extrapolating performance to new applications.

A modified approach using Newton's second law is summarized by Equation 2 as a volume integral of lift force due to the acceleration of air. That integral is over a volume extending sufficiently in all directions from the airfoil so as to allow the surfaces of the control volume to exhibit no pressure or velocity gradients associated with air flow over the airfoil. Other assumptions include neglecting air's buoyancy and flight equilibrium.

$$Lift\ Force = -\iiint ma_z \qquad \text{Equation 2}$$

The integral is substantially the integral of the derivative (dV/dz) of the streamlines (such as those of Figure 1) as weighted by air's density. For an airfoil in *equilibrium flight*, the elegantly simple output of this volume integral is that the net ***ma*** of air for the volume is equal to ***mg*** of the aircraft. Simply stated, air's downward acceleration replaces the aircraft's downward acceleration is satisfaction of gravity's pull.

Equation 2 indicates the following about an airfoil and air's streamlines around that airfoil:

- the streamlines reveal their derivatives by inspection and thus reveal the source of air's acceleration (lift force or force counter to lift),
- acceleration of air occurs at a distance from the wing with air's pressure (laws of continuity) transferring the force from the point of generation to the surface of the wing, and
- an inspection of an airfoil's surfaces, in view of how air must bend to pass by the surfaces, reveals the impact of the surfaces on lift.

Hence, it is not downwash that generates lift; rather, both lift and downwash are the result of air's acceleration/bending due to flow of air around an airfoil.

Relevance to Lift-to-Drag Ratios - The lift-to-drag ratio (L/D) is a critical design feature for aircraft. During *equilibrium flight*, the lift is equal to the weight of the aircraft; and that weight divided by the L/D is the thrust needed to maintain equilibrium flight. For a glider, a similar analysis relates the L/D ratio to the minimum rate at which the glider can descend to achieve maximum flight time.

As a benchmark on L/D ratios, for jets, upper-end gliders, and pure airfoils are 14:1 to 21:1, 70, and 85, respectively.[4] If these values have been historically impacted by paradigms causing incorrect interpretation of how certain surface features impact lift, greater values of each could be attained.

Control Volume Problem-Solving Approaches - The equation 2 axiom resides around a quantitative analysis forces in the control volume around an aircraft. With selection of a control volume where surfaces cancel, the only remaining vertical forces are gravity, the objects acceleration, and air's acceleration. At equilibrium flight the object's acceleration is zero.

Analytical Geometry - Lift force is transferred to an object through surfaces/platforms and can be approximated as the sum of a series of platform contributions plus interferences associated with the joining of the platforms. Platforms contribute lift approximately equal to $A\ \Delta P' \text{ Cos}(\Theta)$, where Θ is degrees of pitch. Here, $\text{Tan}^{-1}(\Theta)$ is the L:D of the platform and the platform's respective $A\ \Delta P'$ is the weighted contribution of that L:D to the whole. (Here $\Delta P'$ equals to average gauge pressure below a platform and average negative gauge pressure above a platform.)

Table 1. Example L:D corresponding to pitch.

Surface Θ (degrees)	L:D of that Surface
0.00	infinity
1.00	57
2.00	29
3.00	19
4.00	14

Based on the values of L:D in Table 1, higher/lower pressures generated from downward bended/accelerated air should be positioned over surfaces of 0°-3° pitch for Terretrane (flying trains with limited wings) to provide an overall L:D of 12:1 or greater. The targeted "lift paths" should have L:D greater than the targeted value to compensate for drag on surfaces that have no lift value. For Terretrans (aircraft), wings can be extended at L:D values near 40:1, and so, a more selective criterion of surfaces of 0°-1.5° pitch is appropriate.

Analysis

The volume integral term of Equation 2 should include all air flow caused by the airfoil being analyzed, and aerodynamic patterns such as vortices substantiate that air undergoes acceleration (direction change) at distances relatively far from the airfoil. Laws of continuity provide mechanisms for transfer of forces to and from the airfoil surfaces, including both pressure waves and air flow (e.g. velocity of downwash).

In view of this behavior, effective airfoil design should include the following three heuristics plus a defining of *lift efficiency*:

1) maximize surfaces that accelerate air downward and minimize surfaces that accelerate air upward,
2) form and preserve "pockets" of lower pressure above airfoil surfaces and "pockets" of higher pressure below airfoil surfaces over flat (or laterally-concave) surfaces at 0°-4° pitch,
3) relax the air pockets gradually so as not to form turbulence or vortices, and.
4) the maximum attainable lift an airfoil can generate in an ideal gas is the longitudinal cross-sectional area times the square of longitudinal velocity where the *lift efficiencies* of an airfoil are actual lifts divided by this maximum lifts. A single *lift efficiency* characterization would be the maximum efficiency at an optimal velocity.

For complex objects (e.g. an aircraft) approximated as multiple airfoils, the overall efficiency is a fraction-weighted addition of the efficiencies minus a factor for interference.

Discussion

Traditional explanations of lift based on Bernoulli's equation and Newton's second law tended to focus on a small part of the big picture. For example, Gilbert simplified the theory[3] to the form of Equation 3.

$$F = ma = \left(\rho \frac{dx}{dt} dy\, dz\right)\left(\frac{dz}{dt}\right) \approx \rho A \varepsilon V_x^2 \qquad \text{Equation 3}$$

This approximation of lift being proportional to velocity squared is only for interactions near surfaces where the longitudinal velocity of the airfoil dominates both the mass flow rate of air and the change in vertical velocity. The implicit nature of front (impacting) and rear (vacuum-driving) surfaces of an airfoil/object on the velocity term of this equation is the origin of the definition of *lift efficiency* (i.e., a definition based on longitudinal cross-sectional area).

At locations a few feet away from the airfoil surfaces, the air acceleration is determined by residual pressure and velocity gradients, rather than airfoil surfaces that stick to air (back side of wing) or block the path of air (front side of wing). And due to this, air accelerations distant from the air foil are typically much lower than near the airfoil.

However, under certain circumstances (e.g. vortices) upward acceleration of air at locations distant from the airfoil can be significant. Spiral vortices tend to form at wing tips; they are vortices around the longitudinal axis as a result of air flowing from higher pressures below the wing to lower pressures above the wing in paths around the tips of wings. Winglets block the path of greatest pressure gradients (the driving force for spiral vortex formation), reduce the magnitude of the vortices, and produce better L/D ratios.[1][5]

Vortices are a form of turbulence. Turbulence tends to have non-negligible upward acceleration and reduces lift. While the Equation 2 volume integral does not predict when turbulence will occur, it quantifies how the turbulence impacts overall lift.

The goal of equation 2 and the four heuristics is to provide rules of thumb to both design surfaces to provide beneficial air acceleration and to preserve resulting pressure gradients.

Application of Heuristic 1 - An inspection of typical aircraft wings and fuselages

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[1] Tumbling vortices along lateral axis can form above airfoils when pitch is too great or ice reduces the interfacial tension between air and the upper surface of the wing. This can reduce lift and cause an aircraft to stall.

reveals that relatively minor modifications in design can result in generation of more downward air momentum (ma). Figure 2 illustrates two example modifications.

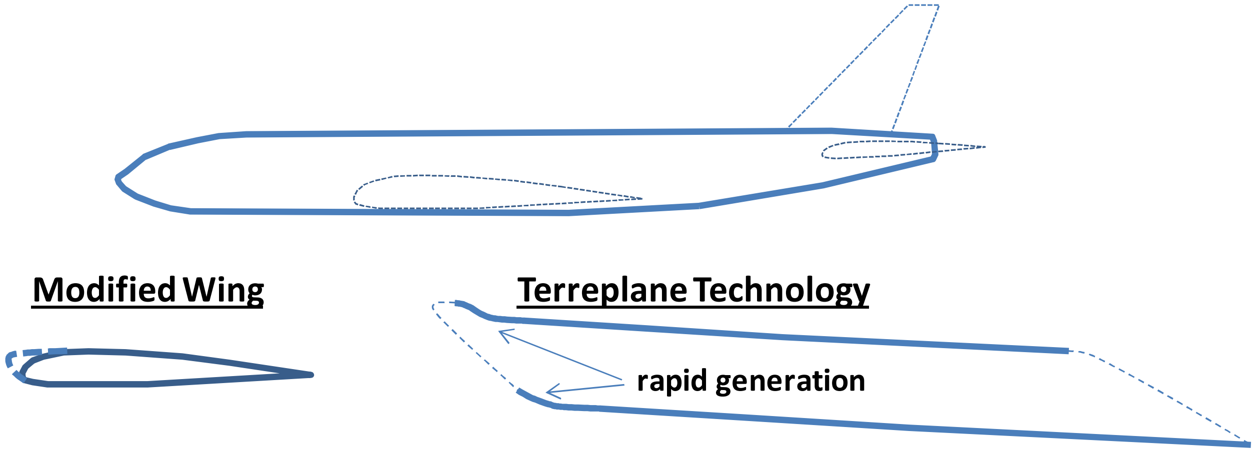

Figure 2. Jet wing and body with modified wing showing higher leading edge (dashed line) and 3-D representation of symbolic fuselage modification showing aerodynamic lift feature consistent with providing more lift-generating air acceleration.

An inspection of the Figure 2 wing airfoil reveals that a higher leading edge of the wing produces greater lift. The typical explanation of lift in terms of high velocity of flow over a wing teaches contrary to this airfoil modification. This has been confirmed experimentally.[6, 7]

Common fuselage types are "flying wing" and "tubular". An inspection of a common tubular fuselage (Figure 2) reveals near zero or even negative *lift efficiency*--the front half is nearly symmetric and the rear half bends more air upward than downward. A possible explanation for this design is that the lift and carrying logistics of most aircraft are separated into wing design and fuselage design, respectively; this is an example of how a paradigm can block the advancement of a technology.

A fuselage's longitudinal cross section will typically be of greater area than the total cross section of the thickest parts of the wings. Modest changes could create different patterns of air acceleration; patterns that could provide lift from fuselage surfaces. A symbolic modification is provided by Figure 2. Modifications would need to include features to preserve the pressures on these surfaces. Heuristic 2 is about preserving and using "pockets" of pressure.

Application of Heuristic 2 - Air is able to flow from the high pressure pocket below an airfoil to the low-pressure pocket above an airfoil by going around the a) side, b) back, and c) front of the airfoil during subsonic flight. Both winglets and increased wing aspect ratios partially block air flow around the sides of wings. Winglets and extended wings (high aspect ratios) are literally "walls" that preserve pockets in manner where the benefit of preserving pressure outweighs the risk of increasing drag.

An application of Heuristic 2 to the fuselage modifications suggest: a) the new surfaces should be the full width of the widest part of the fuselage and b) "walls" (e.g. sideboards) should be placed along the sides of the fuselage's lift-generating surfaces as illustrated by Figure 2.

Application of Heuristic 3 - Heuristic 3 indicates that after passing air has had a reasonable residence time of high (or low) pressure, release of the air should be in a manner that provides for gradual elimination/relaxation of remaining pressure gradients

An example of badly released pressure pockets is downwash. Downwash is a term used to describe air with a downward velocity vector component coming off the back of wings. An undesirable aspect of downwash is that it can lead to a rapid upward acceleration of air behind the wing to fill the space that was occupied by the downwash air. Downwash is not desirable from either the perspective of lift efficiency or impact on tailing aircraft.

A primary method to minimize downwash is to reduce the pitch of an airfoil. In practice, high pitch positions are useful for takeoff and landing, where pitch is used to increase lift at velocities lower than the designed cruise velocity for the airfoils. The pitch used at cruising speeds generates minimal to zero downwash; it is an application of heuristic 3.

[FIGURE NOT AVAILABLE]

Figure 3. Illustration of flying goose where spaces between ends of tail and wing feathers provide a natural diffuser that reduces turbulence such as vortices that form behind and at the edges of airfoils.

Nature reveals other applications of heuristic 3. As illustrated by Figure 3, the tail and wing-end feathers of a Canadian goose are spread with gaps between the feathers. The spaces between feather tips act as diffusers that allow the gradual relaxation of pressure gradients. The flow of air between feathers slows air movement and additional lift is realized from the sheer force of upward flowing air between feathers.

Tethered Glider Applications - The path leading to this paper was not based on the goal to improve the design of jet or prop aircraft. Rather, the path was one of advancing research and development of a new transportation system referred to as Terreplane.[8-10]

Terreplane is based on tethered gliders being pulled by linear motors traveling along zipline-type guideways. The cable guideways are inexpensive, easy to route, and can provide travel velocities faster than the fastest of high speed rail. When cruising, vehicles have full aerodynamic lift so that purely longitudinal forces on the zipline-type guideway both keep the guideway straight and dampen vertical/lateral movement of the cable guideway. A vehicle design challenge is to attain full aerodynamic lift with minimal or no wings to make routing of the guideway easier.

The design approaches for Terreplane fuselages are directly applicable to maximizing the lift of aircraft fuselages. Figure 4 compares a Terreplane tethered glider to an airline where both incorporate design features based on the four heuristics of this paper; the airline has a similar fuselage with a wider wingspan.

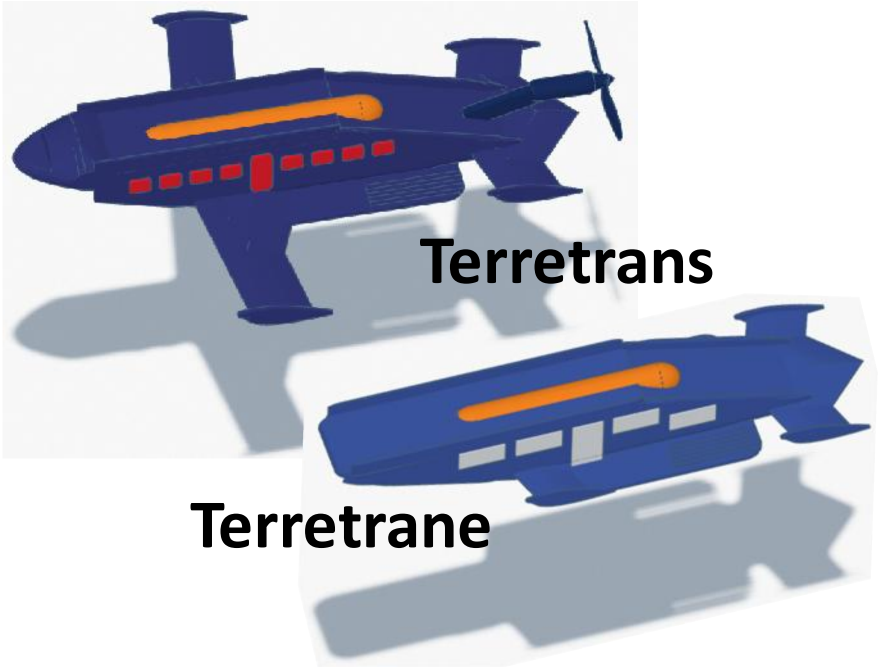

Figure 4. Base case illustrations of airline (engine(s) not shown) and Terreplane tethered glider that incorporate lift features on fuselages. The ratios of wingspan to fuselage width are 5:1 and 2:1 for the Airline and Terreplane glider, respectively.

Terreplane vehicles as directly powered by grid electricity have reduced fuel weights and increased energy efficiency as compared to jet engines; these combine with operational logistics to create the potential for an 80% reducing in passenger-mile fuel consumption (4X increase in fuel economy).[11] The kingpin for realizing this improved fuel economy is the ability to use fuselage surfaces to create lift.

Figure 4 provides base case illustrations (not optimized) of an airline and a Terreplane glider that incorporate lift features on the fuselage. An application of the Bernoulli equation interpretation of lift (based only on velocity) teaches that the fuselages would contribute little to the total lift. Heuristic 4 teaches that the fuselage body can contribute more lift than the wings. As both the wing and fuselage approach the shape of airfoils (combined with application of good design heuristics), L/D ratios should progress from about 14:1 toward 85:1.

Vehicle Control - Positive feedback tends to be an undesirable design feature for many vehicular functions. For example, a slight starboard yaw of an aircraft nose should not lead to an out of control spin or stall (as a result of positive feedback). A torque balance of the tethered glider reveals no control issues relative to positive feedback when the tether is attached sufficiently forward on the vehicle.

The front of the free-flying airline fuselage of Figure 4 could have control issues where a starboard or stern turn (yaw) could result in the simultaneous increase in drag and reduction in lift. Hence, a more-rounded nose is illustrated on the airline than on the Terreplane vehicle. The issues are: a) is the cost for correcting the control issues (or balancing design features) more than the value of increased L/D ratios and b) are alternatives such as flying wing designs better to attain high L/D ratios.

The modified fuselage of the Figure 4 vehicle has two distinct advantages over the flying wing: a) an extended longitudinal dimension (i.e. ratio of wing cord to wing thickness is preferably greater than 5:1) better accommodates a reasonable fuselage height with less of a compromise of the L/D ratio and b) the extended longitudinal dimension enables easier and more effective control features. Today's standard tubular fuselage with zero or negative lift efficiency is certainly not an optimal balance between control and lift. Even modest changes could increase in the near-zero *lift efficiencies* of tubular airline fuselage and have dramatic ramifications to the industry.

Conclusions

An approach of applying a volume integral to air's acceleration around an aircraft yields two inherently meaningful results of: 1) during *equilibrium flight* the net downward "ma" (mass times acceleration) of air is equal to the gravitational force acting on the aircraft and 2) insightful interpretations of lift can be attained by inspection of airfoil surfaces and the manner in which those surface force air upward versus downward.

This work advocates the replace of paradigms with new approaches as follows:

- The paradigm that increased air velocity (with resulting lower pressure) above wings is what causes lift should be replaced with the recognition that lift is created by the manner in which air foils cause downward acceleration of air.
- The paradigm that large wing aspect ratios are needed for high L/D ratios should be replaced with the heuristic that walls (e.g. wider wings, winglets) increase lift by strategically preserving pockets of high/low pressure air.
- The paradigm that a wing's surface area and respective lift coefficient are the best way to characterize lift should be replaced with a lift efficiency based on longitudinal cross-sectional area.

As with any paradigm that has crippled an industry, there are many excuses as to why certain paths of improvement were not pursued. However, good engineering is not about excuses; good engineering is about risk-benefit analyses. Thorough risk-benefit analyses are beyond the scope of this paper. The scope of this paper is to identify a simplified explanation of flight that is simple enough to provide insight into the design process. That explanation includes the presentation of Equation 2, heuristics to assist in applying Equation 2, and the defining of *lift efficiency* as a more-meaningful term for characterizing airfoils and complex aircraft.

Definitions

A: Area, of longitudinal cross section.
equilibrium flight: flight at constant velocity and altitude.
F: force.
gm: gravitational acceleration times mass.
lift efficiency: actual lift of an airfoil (body) divided by longitudinal cross-sectional area times the square of longitudinal velocity.
ma: mass times acceleration.
P: Pressure.

t: time.
V, V_x: velocity, velocity in x direction.
x-axis: horizontal longitudinal axis.
y-axis: horizontal lateral axis.
z-axis: vertical axis.
ε: is the downwash angle.
ρ: density of air.

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